A model for long range wind farm wakes, sensitive to stability

Summary

An analytical model of wake recovery in the gap between offshore wind turbine arrays is presented. The model is sensitive to surface layer stability and easy to apply. Coupled to inter-array wake loss models, this allows for cooperative control between arrays to be investigated.

Introduction

Induction control is a well understood concept in wind energy: setting upwind turbines for less-aggressive energy extraction thereby leaving more energy in the air for downwind turbines. Across an entire array, the aim is to realise a net energy gain. Simple models can be constructed (see e.g. [1]) which indicate the potential for a net energy gain in the context of a single pair of aligned turbines. This also indicates the important role played by stability in moderating the net energy gain potential. Applying simple wake loss models of this type to an entire array provides useful insight into the upper limits of induction control potential energy gains. Real-world accuracy requires more sophisticated models which typically indicate a lesser degree of potential gain.

Increasingly crowded seas, and increasing contention for wind resource, lead naturally to the question of whether cooperative control between large turbine arrays (or between clusters) offshore has potential benefit. In other words, if an upwind array deliberately reduces generation, thus increasing a downwind array's generation, can this be a net gain? This technical note presents a 'top-down' wake loss model for long range wakes which may be applied to this question.

Modelling Approach

The top-down model used is based on the modelling approach of Smith [2]. A 1-dimensional, streamwise version of Smith's model, retaining only forcing and vertical mixing terms, can be written as:

$$U\frac{\partial u_B'}{\partial x} = F_x - Cu_B' \tag{1}$$

Here x is the streamwise distance, U is the depth-averaged wind speed, u_B' is the depth-averaged wind speed perturbation due to a forcing F_x from wind farm drag, and C is the combined Rayleigh friction coefficient. The local solution to this equation with constant forcing, and with integration constant K, is:

$$u_B'(x) = \frac{F_x}{C} + Ke^{-\frac{C}{U}x} \tag{2}$$

Defining upwind (subscript u) and downwind (d) turbine arrays, of lengths L_u and L_d respectively, separated by distance S, we define their respectively constant drag forces (due to turbine drag) as follows:

$$F_x = \begin{cases} F_u, & -L_u \le x \le 0 \\ F_d, & S \le x \le S + L_d \\ 0, & otherwise \end{cases}$$

Assuming zero wind speed perturbation upwind, this allows us to determine a piecewise solution:

$$u_B'(x) = \begin{cases} 0, & x < -L_u \\ \frac{F_u}{C}\left(1 - e^{\frac{-C}{U}(L_u+x)}\right), & -L_u \le x \le 0 \\ \frac{F_u}{C}\left(1 - e^{\frac{-C}{U}(L_u)}\right)e^{-\frac{C}{U}x}, & 0 < x < S \\ \frac{F_d}{C} + \left[\frac{F_u}{C}\left(1 - e^{\frac{-C}{U}(L_u)}\right) - \frac{F_d}{C}e^{\frac{C}{U}S}\right]e^{-\frac{C}{U}x}, & S \le x \le S + L_d \\ \left[\frac{F_d}{C}\left(e^{\frac{C}{U}(S+L_d)} - e^{\frac{C}{U}S}\right) + \frac{F_u}{C}\left(1 - e^{\frac{-C}{U}(L_u)}\right)\right]e^{-\frac{C}{U}x}, & x > S + L_d \end{cases} \quad (3)$$

The solution for the gap between arrays $0 < x < S$ is the key contribution from the Smith model here. The intra-array segments for the upwind ($-L_u \le x \le 0$) and downwind ($S \le x \le S + L_d$) arrays could be replaced by standard wake models easily.

Here, the forcing values are calculated as follows (F_u is shown, F_d is equivalent):

$$F_u = \frac{-\frac{1}{2}\left(U + u_B'\right)^2 C_{T,u} A_{rotor}}{(\Delta x_u)^2 H}$$

Note that Δx is the streamwise and cross-stream spacing between turbines, H is the height of the atmospheric boundary layer and that the turbine drag coefficient $C_{T,u}$ is a function of the local wind speed entering the array, estimated as $U + u_B'$. Equations (3) then provide a direct means to explore wake interaction between arrays separated by distance S, given knowledge of the thrust curve, and the wind and stability conditions. The model is one dimensional, therefore may be considered as representing 'infinitely wide' arrays.

Application Example

Two arrays were considered, separated by 10 km and each 15 turbines deep with a uniform inter-array separation between turbines of 8 rotor diameters. Power and thrust curves for a representative 5 MW wind turbine were used, see Figure 1. Both a 'Baseline' and a 'De-powered' configuration was used.

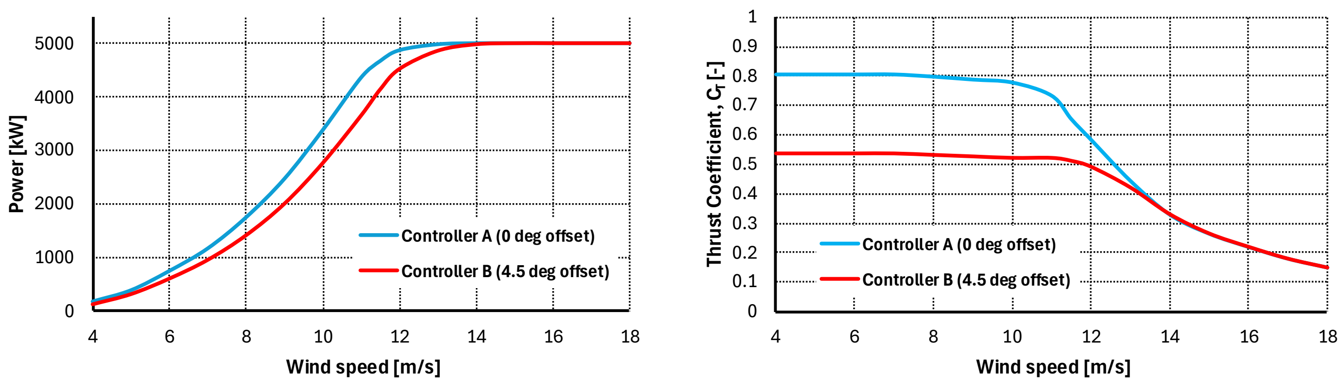

Figure 1: Wind Turbine Power (left) and thrust curves (right)

A case with freestream wind speed of 8 m/s was modelled. Following [3], a value of C = 0.0001 s^{-1} was applied, representing a realistic low turbulence/high stability case. The upwind array was modelled with both the default/baseline and the de-powered control settings, and the downwind array with the default/baseline settings only, in this way exploring 'induction control' potential on a large wind farm scale. The resulting wind speed and turbine power profiles are shown in Figures 2 and 3 respectively. As expected, in the 'de-powered' case the upwind array output is significantly decreased and the downwind output increased: the model results show a 4% overall power gain from applying this cooperation strategy to this case. An otherwise equivalent model configuration with a 'neutral' $C = 0.001\ s^{-1}$ results in an overall power loss of 7% from attempting the same cooperation strategy.

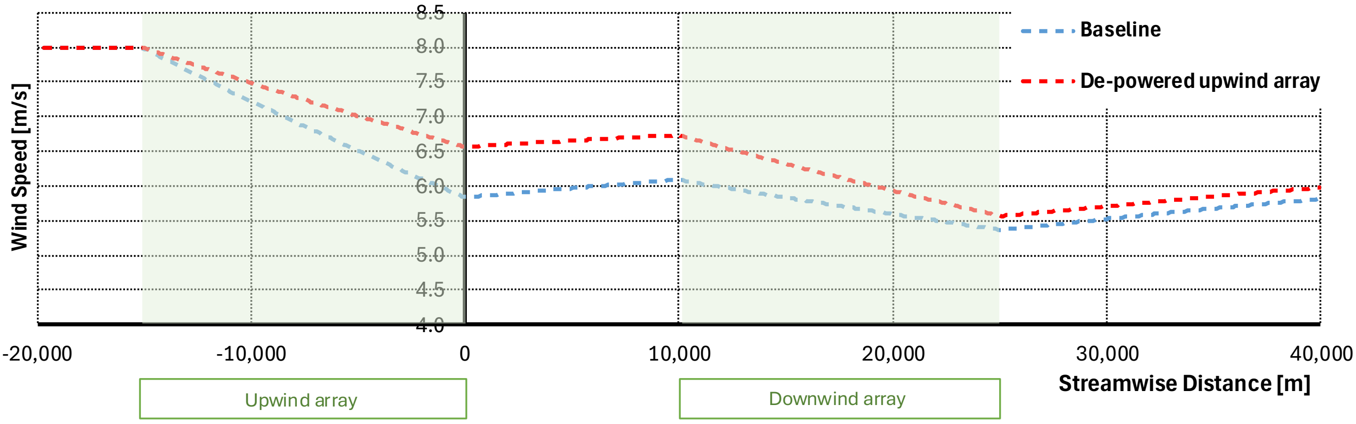

Figure 2: Wind Speed Through Wind Farm – Longitudinal

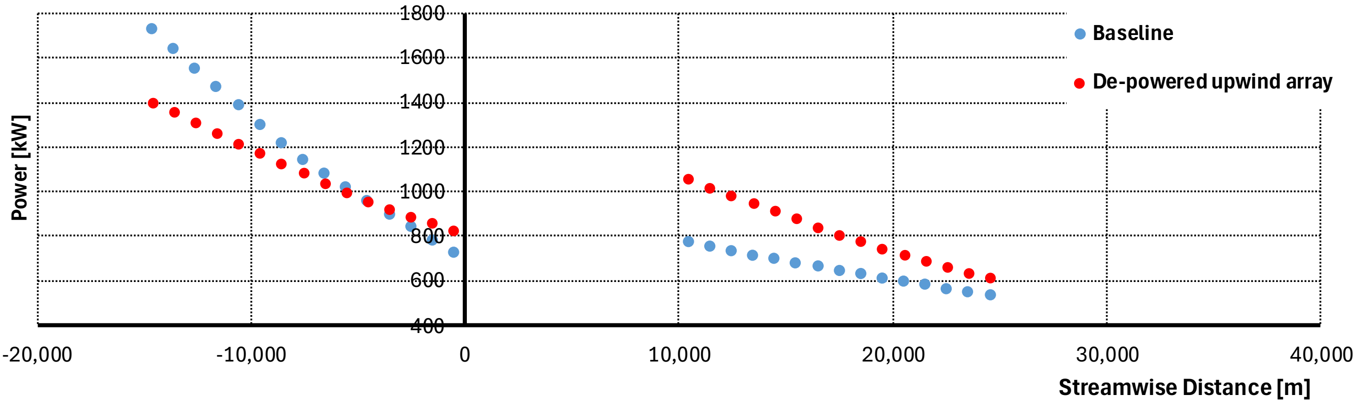

Figure 3: Wind Turbine Powers Through Wind Farm - Longitudinal

Conclusion

The results indicate that cooperative control between offshore wind farms may be beneficial under stable atmospheric conditions. Stability is therefore a key factor in assessing inter-array control strategies. The model presented allows exploration of wind farm cooperative control scenarios very simply, as an additional option alongside comprehensive versions of the Smith model (see e.g. [3],[4]) or to higher-fidelity modelling. It is likely to be useful as a 'first look' tool. Combining with a familiar intra-array wake loss model, which may have a wide validation basis and therefore high confidence, and retaining the present model for the stability-influenced wake recovery in gaps between arrays (inter-array wake loss), would be a worthwhile approach.

References

[1] 'Wind Farm Induction Control Sensitive to Surface Layer Stability', B J Gribben, KBR Technical Journal 2024.

[2] Smith, R.B. Gravity wave effects on wind farm efficiency. *Wind Energ.*, **13**: 449-458, 2009.

[3] Smith, R. B. and Gribben, B. J.: Coriolis recovery of wind farm wakes, Wind Energ. Sci., 11, 233–249, https://doi.org/10.5194/wes-11-233-2026, 2026.

[4] Gribben, B. J.: Gravity Waves and Long-Range Wakes Impacts for Very Large Clusters Offshore, Wind Europe Technology Workshop, Dublin, 2024.